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Introduction

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Macroeconomic Policy Questions and Methods

Monetary policy, fiscal policy, and exchange-rate policy all have powerful effects on the economy. It is not surprising that questions about these macroeconomic policies arise everyday in countries around the world. Sometimes the questions—such as the independence of the central bank, the formation of a currency bloc, or the enforcement of government budget rules—concern the fundamental design of the policy-making institutions. At other times the questions are about implementation of new monetary or fiscal policies—such as how fast to move to a noninflationary monetary policy or how soon to reach a balanced budget. Most frequently the questions concern much shorter-term operational issues, such as whether—in any given week or month—the central bank should be raising or lowering short-term interest rates.

Economists and others—business people, journalists, politicians—are called on, or volunteer, to answer such questions. Of course macroeconomists who work as advisers to the government answer such questions as part of their job. They also have to push their answers through the system until the policy decisions are made. Macroeconomic questions are rarely easy. It seems that the answers are usually either at the cutting edge of economic research—and therefore very controversial—or that there is little research going on and that top research economists are uninterested in the questions. This tends to leave the answering to noneconomists. The questions require quantitative rather than qualitative answers, so that econometric as well as theoretical considerations come into play. Almost always the questions are of great practical importance. Whether the economists' advice is given or not—or taken or not—the resulting policy decisions have profound effects on the performance of the economy.

The purpose of this book is to develop a framework to answer such macroeconomic policy questions. The framework is empirical and can deliver quantitative answers that are usually essential. The framework makes use of modern macroeconomic research, including rational expectations theory, time-consistency analysis, staggered price-setting theories, and new econometric and computer simulation techniques. In fact, the framework *is* modern economic research, and for this reason I think of it as an "interim" approach, recognizing that at least the details—and probably the broad features—of the approach will change as research continues. However, for the present I think that it is a reasonable way to provide "scientific" answers to practical macroeconomic questions. It should not come as a surprise, given its "scientific" aim, that the book is not meant for the casual reader or for the noneconomist. Although it does not dwell on theoretical issues for their own sake, it does require a basic understanding of technical economic and econometric issues.

This chapter develops the overall themes of the book. Section 1.1 outlines three general categories of macroeconomic policy questions. The categories require different modes of policy analysis. I use as examples questions that arose during the early 1990s. Similar questions have arisen before and will undoubtedly arise in the future. The tone of the chapter then shifts abruptly from the practical in Section 1.1 to the technical in Section 1.2, which describes methods used to obtain answers to each category of question. There is no way to avoid this shift, as the technical methods are needed to answer the practical questions, but perhaps the juxtaposition highlights the different levels at which macroeconomists must work. Section 1.3 then uses some stylized examples to illustrate the methods. These same methods are applied to actual policy questions in the remaining chapters of the book as previewed in Section 1.4.

1.1 Policy Rules and Types of Policy Questions

The notion of a policy rule, defined as the systematic response of the policy instruments to the state of the economy, is pervasive in modern macroeconomic research. However, I have found that it is not a common way to think about policy in practice. The distinction between the design of policy rules, the transition to new rules and the operation of policy rules, is meant to help bridge the gap between research and practice. Before explaining this distinction, it is necessary to be precise about the definition of policy rules used in this book.

First, a policy rule is not necessarily a fixed setting for the policy instruments, such as a constant-growth rate rule for the money supply. Feedback rules, in which the money supply responds to changes in unemployment or inflation, are also policy rules. For example, the automatic stabilizers of fiscal policy, such as unemployment compensation and the tax system, can be interpreted as a "policy rule." According to this rule, tax revenues and government expenditures automatically change when the economy expands or contracts. Research on the design of policy rules frequently finds that feedback rules dominate rules with fixed settings for the instruments. This, for example, is the finding of my 1979 *Econometrica* paper. There is little disagreement among macroeconomists that a policy rule should follow this broader interpretation.

Second—and there is more disagreement here—a policy rule need not be a mechanical formula. It can be implemented and operated more informally by policymakers who recognize the general instrument responses that underlie the policy rule and who also recognize that operating the rule requires judgment and cannot be done by computer. This broadens the scope of a policy rule significantly and permits the consideration of issues that would be excluded under the narrower definition. Thus, a policy rule would include a nominal-income rule, in which the central bank takes actions to keep nominal income on target, but it would not include pure discretionary policy. In broadening the definition we need to be careful not to lose the concept entirely. Under pure discretion, the settings for the instruments of policy are determined from scratch each period with no attempt to follow a reasonably well-defined contingency plan for the future. A precise analytical distinction between policy rules and discretion can be drawn from the time-consistency literature. In the time-consistency literature[1] (see Kydland and Prescott [1977], Calvo [1978], or Blanchard and Fischer [1989]), a policy rule is referred to as either the "optimal" or the "precommitted" solution to a dynamic optimization problem. Discretionary policy is referred to as the "inconsistent" or "shortsighted" solution. The advantage of rules over discretion, which the literature amply demonstrates, is one of the reasons why I have focussed on policy rules in this normative-oriented policy research.[2]

Third, if a policy rule is to have any meaning, it must be in place for a reasonably long period of time. For a macroeconomic policy rule, several business cycles would certainly be sufficient, but, for many purposes, several years would do just as well. Policymakers need to make a commitment to stay with the rule if they are to gain the advantages of credibility associated with it. Credibility enhances the performance of an economy under a good policy rule. For example, a credible monetary policy can reduce inflation with less loss of real output. Moreover, if economic analysis is to have much hope of assessing how the economy will perform with a policy rule, some durability of the rule is obviously required. For example, one of our tasks is to calculate how parameters of a reduced-form model change when the parameters of the policy rule change. Such calculations are of little use if the policy rule

[1] For an elementary discussion of the concept of time inconsistency in macroeconomics, see Hall and Taylor (1993, 537–543).

[2] It is not the only reason, however. My research on policy rules predates the time-inconsistency literature and goes back to my undergraduate thesis at Princeton in 1968 and my Ph.D. thesis at Stanford in 1973. Arguments made by Friedman (1948), Phillips (1954), and Lucas (1976) as well as rational expectations per se are other reasons to focus on policy rules.

is constantly in flux. On the other hand, a reasonably long period of time does not mean forever. One can easily imagine technological changes, such as, for example, the development of automatic teller machines, affecting the demand for money, which call for revisions in policy rules.

Now consider the three categories of policy issues, namely, those that relate to: (1) appropriate *design* of a policy rule; (2) the *transition* to a new policy rule once it is designed; and (3) the day-to-day *operation* of a policy rule once it is in place.

Design Questions

The first category pertains to the appropriate design of a policy rule. I give two examples of design questions. Both refer to monetary policy. One pertains to international monetary policy and to the exchange-rate system. The other pertains to domestic monetary policy and to the degree of accommodation by the monetary authorities.

Example: International Monetary Reform. Preparations usually get underway well in advance of the annual economic summit meeting of the leaders of the G-7 countries—Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. In setting up the agenda for summit meetings in the early 1990s, representatives began to suggest that reform of the international monetary system be placed on the agenda. The emergence of new market economies in Eastern Europe and in the former Soviet Union, as well as the possibility of new roles for the International Monetary Fund (IMF) and the World Bank, have given new impetus to such an agenda. One such reform might be the adoption of a system with greater exchange-rate management, perhaps ultimately tying together the dollar, the yen, and the currencies in the European exchange-rate mechanism in a world fixed exchange-rate system.

Questions to address. How should the United States and other countries react to such suggestions? Although the eventual answer would involve political and strategic issues, the underlying economic question is straightforward to state: Would it be a net benefit (quantitatively speaking) for the world economy if the dollar, the yen, and the currencies in the European exchange-rate mechanism were tied together in a world fixed exchange-rate system?

Example: Domestic Monetary Policy Accommodation. In early August 1990, Iraq had just invaded Kuwait. The price of oil rose rapidly, and consumer confidence was dropping. The U.S. economy had been growing slowly since early 1989, partly as a result of a relatively tight monetary policy.

Questions that arose at the time. How accommodative should monetary policy be to this oil-price shock? That is, by how much, if at all, should the instruments of monetary policy adjust in response to this shock? Is the "rule" that the Federal Reserve System (the Fed) appears to be following appropriate? What about other central banks? Should the issue be raised in the next international policy coordination meeting at the OECD in September? Should diplomatic pressure be applied? How much impact will this have on the performance of the U.S. economy? By how much will inflation rise under alternative circumstances?

Transition Questions

This category of questions involves how, when, or how rapidly to implement a new or modified policy rule. I give three examples of questions in this category. Two involve domestic policy, the other international policy.

Example: A Disinflation Path. There is general agreement among economists that a monetary policy rule should be designed to achieve price stability or near-zero inflation. When inflation persists at a higher rate than desired, whether in double digits as in the United States during the late 1970s or around 5 percent as in the late 1980s, the monetary policy rule must be changed so as to achieve a lower inflation.

Questions to address. How rapidly should the new rule be implemented? How quickly should the rate of inflation be reduced? Is a cold turkey or a gradual reduction more appropriate? How can the adverse effects of disinflation on the economy be minimized?

Example: A Path for Structural Budget-Deficit Reduction. In mid-1990, economic growth in the United States was weak, and economists were beginning to forecast a recession. Also, there was increasing evidence that the U.S. structural budget deficit was no longer declining through the Gramm-Rudman law, and negotiations on a multiyear budget agreement were planned. A key goal in entering the budget negotiations was to reduce the structural budget deficit, ideally to near zero. The intent was to change the "rule" for fiscal policy so that the budget would be balanced at full employment rather than in deficit.

Questions that arose at the time. By *how much* is it appropriate to reduce the Federal budget deficit in the first year of a multiyear agreement? One percent of the GNP? More? Less? Should the focus be on the structural deficit, with the actual deficit permitted to increase if there was a full-blown recession (as there turned out to be)? What would this distinction between structural and actual deficit do to the credibility of the deficit-reduction plan? What should the Fed do in response to a fiscal contraction brought on by a reduction in the federal budget deficit? By *how much* should the Federal funds rate be reduced? Does it matter whether the reduction in the deficit is credible?

Example: International Coordination to Reduce Saving-Investment Gaps. As part of a series of bilateral talks (called the Structural Impediments Initiative), the government of Japan decided in mid-1990 to shift its fiscal policy stance so as to increase the level of public infrastructure investment as a share of GNP. A ten-year plan was proposed. The goal was to reduce the gap between savings and investment and thereby to reduce the trade surplus in Japan and, it was hoped, to reduce trade frictions between Japan and the United States. In this respect, the goal reflected the accounting identity that the gap between saving and investment equals the trade surplus.

Questions that arose at the time. By how much is it appropriate to raise public infrastructure investment in the first year of the plan? Does it matter that the Japanese economy is booming and that the Bank of Japan is raising interest rates? Is there any chance that the change would show up in a quantitatively significant effect on the trade surplus in the first year and thereby start to reduce pressures for trade restrictions immediately?

Operational Questions

This category pertains to the day-to-day operation of a policy rule. Suppose that the Fed's policy rule is to raise systematically interest rates when inflation rises or the economy booms and to lower interest rates when inflation falls or the economy slumps. A typical example of how to operate such a rule concerns the interpretation of current developments. Here is an example that pertains to developments in international capital markets.

Example: The Source of a Rise in Long-Term Interest Rates. In late 1989 and early 1990, long-term interest rates were rising sharply. Two explanations were frequently offered by analysts. (1) Economic unification of East and West Germany was likely and was expected to raise the demand for capital, and/or increase the budget deficit in Germany. Expectations of higher future interest rates in Germany were raising long-term interest rates in Germany and other countries, including the United States. (2) Renewed inflationary pressures were being reflected in higher long-term interest rates.

Questions that arose at the time. Is the rise in interest rates due to developments in Eastern Europe, and in particular, to the expectation that the budget deficit will soon be increasing in Germany? Or is the increase due to additional pressures on inflation and the expectation of increases in future inflation? If the former is true, then the Fed would be true to the operation of its policy rule if it continued to lower interest rates as it had been planning to do in a weak economy with apparently declining inflationary pressures. Otherwise, it should hold off on further declines in interest rates. Answering such questions is clearly essential for the effective operation of a policy rule.

1.2 Technical Preliminaries: Stochastic Modeling with Rational Expectations

It must be difficult for a noneconomist to imagine how an economist could answer any of these questions without a quantitative framework. Although back-of-the-envelope calculations or textbook diagrams might provide intuition or help explain the answer, they don't tell the economist how to balance off hundreds of interrelated factors that bear on the answer. Moreover, modeling expectations in a reasonably sophisticated way is essential. The questions involve expectations of future interest rates, government spending, exchange rates, and oil prices.

Given the current state of economic knowledge, the most sophisticated quantitative model of expectations is a rational expectations econometric model, and this is the type of model I use in this book to answer these questions. In order to use a rational expectations model, one has to be able to solve it and understand what the solution means. Most rational expectations models that are useful for practical applications are either large or nonlinear or both. Numerical methods are needed to solve them. In order to understand how these methods deliver answers to the policy questions, there is no alternative to studying their technical properties, and this is the objective of this section.

I start with the most elementary of all stochastic rational expectations models: a linear relation between one variable, one expectation, and one stochastic shock. The solution method is one that can be used in different modes to handle the different categories of questions. I consider the solution of this model in some detail. It handles many stylized economic problems. Making analogies with this simple model, I then discuss briefly how one handles larger and nonlinear models.

Linear Models with One Variable

Let y_t be a variable satisfying the relationship

$$y_t = \alpha E_t y_{t+1} + \delta u_t, \tag{1.1}$$

where α and δ are parameters and E_t is the conditional expectation based on all information through period t, including knowledge of the model. The variable u_t is an exogenous shift variable or "shock" to the equation. It is assumed to follow a general linear process with the representation

$$u_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}, \tag{1.2}$$

where θ_i, $i = 0, 1, 2, \ldots$, is a sequence of parameters, and where ε_t is a serially uncorrelated random variable with zero mean. Note that ε_{t+i} for

$i \leqslant 0$ is in the information set for making the conditional expectation E_t. The shift variable could represent a policy variable. Alternatively it could represent a stochastic error term as in an econometric equation. In the latter case, δ would normally be set to 1.

The information upon which the expectation in Equation (1.1) is conditioned includes past and current observations on ε_t as well as the values of α, δ, and θ_i. Solving the model means finding a stochastic process for the random variable y_t that satisfies Equation (1.1). The forecasts generated by this process will then be equal to the expectations that appear in the model. In this sense, expectations are consistent with the model, or equivalently, expectations are rational.

The technical discussion will focus on a particular macroeconomic interpretation of Equation (1.1): an elementary macroeconomic model with perfectly flexible prices. This model is of course, far simpler than the econometric models needed to address the questions in Section 1.1. Nevertheless, the technical issues are conceptually the same. There is one policy instrument (the money supply) and one target variable (the price level). For this type of model the real rate of interest and real output are unaffected by monetary policy and thus they can be considered fixed constants. The demand for real-money balances—normally a function of nominal interest rate and real output—is therefore a function of only one variable: the expected inflation rate. If p_t is the log of the price level and m_t is the log of the money supply, then the demand for real money can be represented as

$$m_t - p_t = -\beta(E_t p_{t+1} - p_t), \tag{1.3}$$

where β is a positive coefficient. In other words, the demand for real-money balances depends negatively on the expected rate of inflation, as approximated by the expected first difference of the log of the price level. Equation (1.3) can be written in the form of Equation (1.1) by setting $\alpha = \beta/(1 + \beta)$ and $\delta = 1/(1 + \beta)$ and by letting $y_t = p_t$ and $u_t = m_t$. In this example, the variable u_t represents the supply of money. The money supply is assumed to be generated by the process of Equation (1.2).

The stochastic process for the shock variable u_t is assumed in Equation (1.2) to have a very general form. Any stationary stochastic process can be written this way. If u_t is a policy variable, then one can consider the design of alternative policy rules—as one would do to answer policy-design questions—by specifying different stochastic processes for u_t. For example, one could also have u_t be a function of y_t, which would entail a reinterpretation of the parameters in Equation (1.1).

In both *implementation* and *operation* applications, one is interested in "experiments" in which the policy variable is shifted in a special way and the response of the endogenous variables is examined. In forward-looking rational expectations models, the response depends not only on whether the shift in the policy variable is temporary or permanent but also on whether it is credibly anticipated or unanticipated. For example, the impact of future reductions in the budget deficit discussed in Section 1.1 would depend on whether the reductions were anticipated by the markets. Equation (1.2) can be given a special interpretation to characterize these different experiments, as follows:

Temporary versus permanent shocks. The shock u_t is purely temporary when $\theta_0 = 1$ and $\theta_i = 0$ for $i > 0$. Then any shock u_t is expected to disappear in the period immediately after it has occurred, that is, $E_t u_{t+i} = 0$ for $i > 0$ at every realization u_t. At the other extreme, the shock u_t is permanent when $\theta_i = 1$ for $i \geqslant 0$. Then any shock u_t is expected to remain forever, that is, $E_t u_{t+i} = u_t$ for $i > 0$ at every realization of u_t. By setting $\theta_i = \rho^i$, a range of intermediate-persistence assumptions can be modeled as ρ varies from 0 to 1. For $0 < \rho < 1$ the shock u_t phases out geometrically. In this case u_t can also be interpreted as $u_t = \rho u_{t-1} + \varepsilon_t$, a first-order autoregressive model.

Anticipated versus unanticipated shocks. Time delays between the realization of the shock and its incorporation in the current information set can be introduced by setting $\theta_i = 0$ for values of i up to the length of time of anticipation. For example, we can set $\theta_0 = 0$, $\theta_1 = 1$, $\theta_i = 0$ for $i > 1$, so that $u_t = \varepsilon_{t-1}$. This would characterize a temporary shock that is anticipated one period in advance. In other words, the expectation of u_{t+1} at time t is equal to u_{t+1} because $\varepsilon_t = u_{t+1}$ is in the information set at time t. More generally, a temporary shock anticipated k periods in advance would be represented by $u_t = \varepsilon_{t-k}$. A shock anticipated k periods in advance and that is then expected to phase out gradually would be modeled by setting $\theta_i = 0$ for $i = 1, \ldots, k-1$ and $\theta_i = \rho^{i-k}$ for $i = k, k+1, \ldots$, with $0 < \rho < 1$.

Solving the Model: Unanticipated Shocks. In order to find a solution for y_t, we begin by representing y_t in the unrestricted infinite moving average form

$$y_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}. \tag{1.4}$$

Finding a solution for y_t requires determining values for the undetermined coefficients γ_i such that Equations (1.1) and (1.2) are satisfied. Equation (1.4) states that y_t is a general function of all possible events that may potentially influence y_t.

Note that the solution for y_t in Equation (1.4) is a general stationary stochastic process. From Equation (1.4), one can easily compute the variance and the autocovariance function of y_t and thereby study the effects of different policy rules on the stochastic behavior of y_t. That is, one can study the design of policy rules. But one can also use Equation (1.4) to calculate the effect of a one-time unit shock to ε_t, that is, experiment with implementation and operation of policy rules. The dynamic impact of such a shock is simply $dy_{t+s}/d\varepsilon_t = \gamma_s$.

To find the unknown coefficients, substitute for y_t and $E_t y_{t+1}$ in (1.1) by using (1.4) and solve γ_i in terms of α, δ, and θ_i. The conditional expectation $E_t y_{t+1}$ is obtained by leading (1.4) by one period and by taking expectations, making use of the equalities $E_t\varepsilon_{t+i} = 0$ for $i > 0$ and $E_t\varepsilon_{t+i} = \varepsilon_{t+i}$ for $i \leqslant 0$. The first equality follows from the assumption that ε_t has a zero unconditional mean and is uncorrelated; the second follows from the fact that ε_{t+i} for $i \leqslant 0$ is in the conditioning set at time t. The conditional expectation is

$$E_t y_{t+1} = \sum_{i=1}^{\infty} \gamma_i \varepsilon_{t-i+1}. \tag{1.5}$$

Substituting (1.2), (1.4), and (1.5) into (1.1) results in

$$\sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} = \alpha \sum_{i=1}^{\infty} \gamma_i \varepsilon_{t-i+1} + \delta \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}. \tag{1.6}$$

Equating the coefficients of ε_t, ε_{t-1}, ε_{t-2} on both sides of the equality (1.6) results in the set of equations

$$\gamma_i = \alpha\gamma_{i+1} + \delta\theta_i \qquad i = 0, 1, 2, \ldots. \tag{1.7}$$

The first equation in (1.7) for $i = 0$ equates the coefficients of ε_t on both sides of (1.6); the second equation similarly equates the coefficient for ε_{t-1} and so on.

Note that Equation (1.7) is a deterministic difference equation in the γ_i coefficients with θ_i as an exogenous variable. This deterministic difference equation has the same structure as the stochastic difference Equation (1.1). Once Equation (1.7) is solved for the γ_i, the solution for y_t follows immediately from Equation (1.4). Hence, the problem of solving a *stochastic* difference equation with conditional expectations of future variables has been converted into a problem of solving a *deterministic* difference equation.

Consider first the most elementary case where $u_t = \varepsilon_t$. This is the case of unanticipated shocks that are temporary. Then Equation (1.7) can be written

$$\gamma_0 = \alpha\gamma_1 + \delta \tag{1.8}$$

$$\gamma_{i+1} = \frac{1}{\alpha}\gamma_i \qquad i = 1, 2, \ldots. \tag{1.9}$$

From Equation (1.9) all the γ_i for $i > 1$ can be obtained once we have γ_1. However Equation (1.8) gives only one equation in the two unknowns γ_0 and γ_1. Hence, without further information, we cannot determine the γ_i-coefficients uniquely. The number of unknowns is one greater than the number of equations. This indeterminacy is what leads to nonuniqueness in rational expectations models.

If $|\alpha| \leqslant 1$, then the requirement that y_t is a stationary process will be sufficient to yield a unique solution. To see this, suppose that $\gamma_1 \neq 0$. Since

Equation (1.9) is an unstable difference equation, the γ_i coefficients will explode as i gets large. But then y_t would not be a stationary stochastic process. The only value for γ_1 that will prevent γ_i from exploding is $\gamma_1 = 0$. From Equation (1.9) this in turn implies that $\gamma_i = 0$ for all $i > 1$. From Equation (1.8) we then have that $\gamma_0 = \delta$. Hence, the unique stationary solution is simply $y_t = \delta\varepsilon_t$. In the case of the money-demand Equation (1.3), the price satisfies $p_t = (1 + \beta)^{-1} m_t$.

Using this relationship between the price level and the money supply, the variance of the price level can easily be computed as a function of the variance of the serially uncorrelated money shocks. Thus, the impact of this simple "policy rule" (purely random money shocks) on "macroeconomic performance" can be evaluated, though both are very trivial in this case.

Because $\beta > 0$, a temporary unanticipated increase in the money supply increases the price level by less than the increase in money. This is due to the fact that the log of the price level is expected to decrease to its normal value (zero) next period, thereby generating an expected deflation. The expected deflation increases the demand for money, so that real balances must increase. Hence, the price p_t rises by less than m_t. This is illustrated in Figure 1-1a.

For the more general case where shifts in u_t are expected to phase out gradually, we set $\theta_i = \rho^i$, where $\rho < 1$. Equation (1.7) then becomes

$$\gamma_{i+1} = \frac{1}{\alpha}\gamma_i - \frac{\delta\rho^i}{\alpha} \qquad i = 0, 1, 2, 3, \ldots. \tag{1.10}$$

Again, this is a standard deterministic difference equation. In this more general case, we can obtain the solution γ_i by deriving the solution to the homogeneous part $\gamma_i^{(H)}$ and the particular solution to the nonhomogeneous part $\gamma_i^{(P)}$.

The solution to (1.10) is the sum of the homogeneous solution and of the particular solution $\gamma_i = \gamma_i^{(H)} + \gamma_i^{(P)}$. (See Baumol [1970], for example, for a description of this solution technique for deterministic difference equations.) The homogeneous part is

$$\gamma_{i+1}^{(H)} = \frac{1}{\alpha}\gamma_i^{(H)} \qquad i = 0, 1, 2, \ldots, \tag{1.11}$$

with solution $\gamma_{i+1}^{(H)} = (1/\alpha)^{i+1}\gamma_0^{(H)}$. As in the earlier discussion, if $|\alpha| < 1$, stationarity requires that $\gamma_0^{(H)} = 0$. For any other value of $\gamma_0^{(H)}$ the homogeneous solution will explode. Stationarity therefore implies that $\gamma_i^{(H)} = 0$ for $i = 0, 1, 2, \ldots$.

To find the particular solution, we substitute $\gamma_i^{(P)} = hb^i$ into (1.10) and solve for the unknown coefficients h and b. This gives:

$$\begin{aligned} b &= \rho, \\ h &= \delta(1 - \alpha\rho)^{-1}. \end{aligned} \tag{1.12}$$

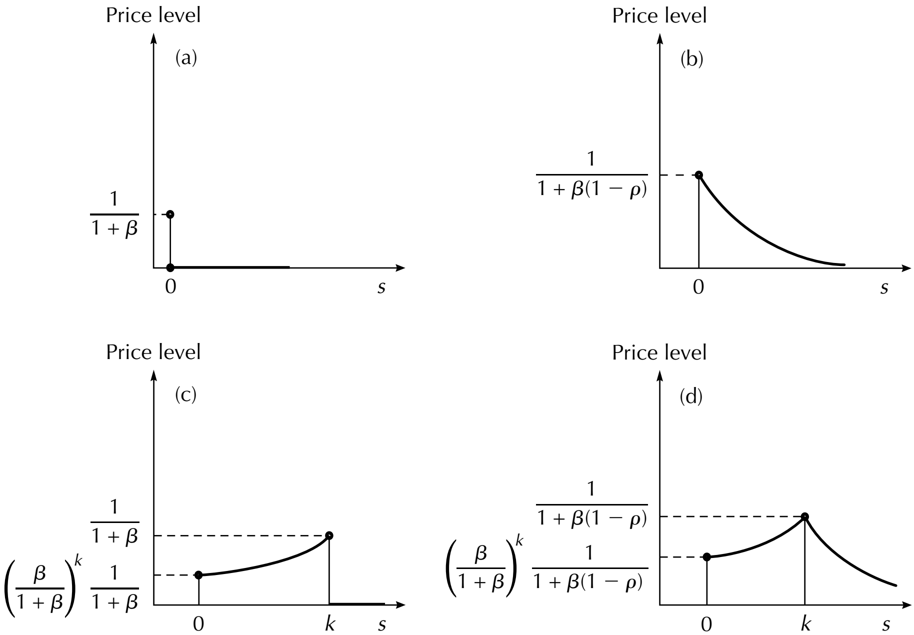

FIGURE 1-1 The Effects of One-Time Shocks in Money-Demand Models. Each panel shows the price-level effect. Panel (a) shows the effect of an unanticipated unit increase in m_t that lasts for one period. Panel (b) shows the price-level effect of an unanticipated increase in m_t that is phased out gradually. Panel (c) shows the price-level effect of an anticipated unit increase in m_{t+k} that lasts for one period. The increase is anticipated k periods in advance. Finally, Panel (d) shows the price-level effect of an anticipated unit increase in m_{t+k} that is phased out gradually. The increase is anticipated k periods in advance.

Because the homogeneous solution is identically equal to zero, the sum of the homogeneous and the particular solutions is simply

$$\gamma_i = \frac{\delta\rho^i}{1-\alpha\rho}, \qquad i = 0, 1, 2, \ldots. \tag{1.13}$$

In terms of the representation for y_t this means that

$$\begin{aligned} y_t &= \frac{\delta}{1-\alpha\rho}\sum_{i=0}^{\infty}\rho^i \varepsilon_{t-i} \\ &= \frac{\delta}{1-\alpha\rho}u_t. \end{aligned} \tag{1.14}$$

For the simple macroeconomic example, this implies that the price level satisfies

$$p_t = \left(\frac{1}{1+\beta(1-\rho)}\right) m_t. \tag{1.15}$$

As in the simpler case, the stochastic properties of the price level can be computed once the value of ρ and hence the stochastic properties of the money supply is known, and the effects of one-time shocks can also be evaluated from these expressions. As long as $\rho < 1$, the increase in the price level will be less than the increase in the money supply. The dynamic impact on p_t of a unit shock to the money supply is shown in Figure 1-1b. The price level increases by less than the increase in the money supply because of the expected deflation that occurs as the price level gradually returns to its equilibrium value of 0. The expected deflation causes an increase in the demand for real-money balances that is satisfied by having the price level rise less than the money supply. For the special case that $\rho = 1$—a permanent increase in the money supply—the price level moves proportionately to money supply as in the simple-quantity theory. In that case, there is no change in the expected rate of inflation since the price level remains at its new level.

If $|\alpha| > 1$, then simply requiring y_t to be a stationary process will not yield a unique solution. In this case, Equation (1.9) is stable, and any value of γ_1 will give a stationary time series. There is a continuum of solutions, and it is necessary to place additional restrictions on the model if one wants to obtain a unique solution for the γ_i. There does not seem to be any completely satisfactory approach to take in this case. One possibility raised by Taylor (1977) is to require that the process for y_t have a minimum variance. An alternative rule for selecting a solution was proposed by McCallum (1983) and is called the "minimum state variable technique." In this case a representation for y_t that involves the smallest number of ε_t terms is chosen, hence giving $y_t = \delta\varepsilon_t$. Fortunately, for the estimated values of the parameters in the empirical models used in this book, this situation never arises.

Solving the Model: Anticipated Shocks. Consider now the case where the shock is anticipated k periods in advance and is purely temporary. That is, $u_t = \varepsilon_{t-k}$, so that $\theta_k = 1$ and $\theta_i = 0$ for $i \neq k$. The difference equations in the unknown parameters can be written as:

$$\begin{aligned}
\gamma_i &= \alpha\gamma_{i+1} && i = 0, 1, 2, \ldots, k-1 \\
\gamma_{k+1} &= \frac{1}{\alpha}\gamma_k - \frac{\delta}{\alpha} && \\
\gamma_{i+1} &= \frac{1}{\alpha}\gamma_i && i = k+1, k+2, \ldots.
\end{aligned} \tag{1.16}$$

The last set of equations in (1.16) is identical in form to Equation (1.9), except that the initial condition is at $k+1$ rather than at 1. For a stationarity condition we therefore require that $\gamma_{k+1} = 0$. This implies that $\gamma_k = \delta$. The remaining coefficients are $\gamma_i = \delta\alpha^{k-i}$, $i = 0, 1, 2, \ldots, k-1$. Hence,

$$y_t = \delta\left[\alpha^k \varepsilon_t + \alpha^{k-1}\varepsilon_{t-1} + \cdots + \alpha\varepsilon_{t-(k-1)} + \varepsilon_{t-k}\right]. \tag{1.17}$$

In the simple macroeconomic example, when a temporary increase in the money supply is anticipated, the price level "jumps" at the date of announcement and then gradually increases until the money supply *does* increase. This is shown in Figure 1-1c. The eventual increase in the price level is the same as in the unanticipated case.

Finally, we consider the case where the shock is anticipated, but is expected to be permanent or to phase out gradually. Then $\theta_i = 0$ for $i = 1, \ldots, k-1$ and $\theta_i = \rho^{i-k}$ for $i \geqslant k$. Equation (1.7) becomes

$$\begin{aligned} \gamma_i &= \alpha\gamma_{i+1} && i = 0, 1, 2, \ldots, k-1, \\ \gamma_{i+1} &= \frac{1}{\alpha}\gamma_i - \frac{\delta\rho^{i-k}}{\alpha} && i = k, k+1, \ldots. \end{aligned} \tag{1.18}$$

The coefficients can be obtained from (1.18) as in the previous cases. The solution for y_t is

$$\begin{aligned} y_t = \frac{\delta}{1-\alpha\rho}\big(&\alpha^k \varepsilon_t + \alpha^{k-1}\varepsilon_{t-1} + \cdots + \alpha\varepsilon_{t-(k-1)} + \varepsilon_{t-k} \\ &+ \rho\varepsilon_{t-(k-1)} + \rho^2\varepsilon_{t-(k-2)} + \cdots\big). \end{aligned} \tag{1.19}$$

For the simple macroeconomic model, where y_t is the price level, the effect of this type of shock is shown in Figure 1-1d. As before, the anticipation of an increase in the money supply causes the price level to jump. The price level then increases gradually until the increase in money actually occurs. During the period before the actual increase in money, the level of real balances is below equilibrium because of the expected inflation. The initial increase becomes larger as the phase-out parameter ρ gets larger. For the permanent case where $\rho = 1$, the price level eventually increases by the same amount as the money supply.

Linear Models with More than One Variable

The above solution method can be generalized and applied to linear models with many endogenous variables. To see this, first note that the simple model in Equation (1.1) can be generalized into a multivariate linear rational expectations model written as

$$B_0 y_t + B_1 y_{t-1} + \cdots + B_p y_{t-p} + A_1 E_t y_{t+1} + \cdots + A_q E_t y_{t+q} = C u_t, \tag{1.20}$$

where the A_i and B_i are matrices, y_t is a vector of endogenous variables, and u_t is a vector of shocks. Equation (1.20) can be made to look much like Equation (1.1) by writing it as

$$E_t z_{t+1} = A z_t + D u_t \tag{1.21}$$

by stacking $y_{t+q-1}, y_{t+q-2}, \ldots, y_{t-p}$ into vector z_t. Equation (1.21) can be solved using matrix generalizations of the method used to solve (1.1). In Equation (1.21), z_t is an n-dimensional vector and u_t is an m-dimensional vector of stochastic disturbances. The matrix A is n by n and the matrix D is n by m.

We describe the solution for the case of unanticipated temporary shocks: $u_t = \varepsilon_t$, where ε_t is a serially uncorrelated vector with a zero mean. The solution for z_t can be written in the general form:

$$z_t = \sum_{i=0}^{\infty} \Gamma_i \varepsilon_{t-i}, \tag{1.22}$$

where the Γ_i values are the n by m matrices of unknown coefficients. Substituting (1.22) into (1.21) and equating the coefficients of the ε_{t-i}, we get

$$\begin{aligned} \Gamma_1 &= A\Gamma_0 + D, \\ \Gamma_{i+1} &= A\Gamma_i \qquad i = 1, 2, \ldots . \end{aligned} \tag{1.23}$$

Note that these matrix difference equations hold for each column of Γ_i separately, that is,

$$\begin{aligned} \gamma_1 &= A\gamma_0 + d, \\ \gamma_{i+1} &= A\gamma_i \qquad i = 1, 2, \ldots, \end{aligned} \tag{1.24}$$

where γ_i is any one of the n by 1 column vectors in Γ_i, and where d is the corresponding column of D. Equation (1.24) is a deterministic first-order vector difference equation analogous to the stochastic difference equation in (1.21). The solution for the Γ_i is obtained by solving for each of the columns of Γ_i separately using (1.24).

The analogy from the one-variable case is now clear. There are n equations in the first vector equation of (1.24). In a given application we will know some of the elements of γ_0 but not all of them. Hence, there will generally be more than n unknowns in (1.24). The number of unknowns is $2n - k$, where k is the number of values of γ_0 we know.

To get a unique solution in the general case, we therefore need $(2n - k) - n = n - k$ additional equations. These additional equations can be obtained by requiring that the solution for y_t be stationary or equivalently, in this context, that the γ_i do not explode. If there are exactly $n - k$

distinct roots of A that are greater than 1 in modulus, then we have exactly the number of additional equations necessary for a solution. If there are less than $n-k$ roots, then we have a nonuniqueness problem.

Suppose this root condition for uniqueness is satisfied. Let the $n-k$ roots of A that are greater than 1 in modulus be $\lambda_1, \ldots, \lambda_{n-k}$. Diagonalize A as $H^{-1}\Lambda H = A$. Then the second equation in (1.24) can be written as

$$H\gamma_{i+1} = \Lambda H\gamma_i \qquad i = 1, 2, \ldots. \tag{1.25}$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} \gamma_{i+1}^{(1)} \\ \gamma_{i+1}^{(2)} \end{pmatrix} = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} \gamma_i^{(1)} \\ \gamma_i^{(2)} \end{pmatrix} \qquad i = 1, 2, \ldots, \tag{1.26}$$

where Λ_1 is a diagonal matrix with all the unstable roots on the diagonal. The γ vectors are partitioned accordingly, and the rows (H_{11}, H_{12}) of H are the characteristic vectors associated with the unstable roots. Thus, for stability we require

$$H_{11}\gamma_1^{(1)} + H_{12}\gamma_1^{(2)} = 0, \tag{1.27}$$

which implies that the solution of the unstable part of the system stays at zero. Equation (1.27) gives the additional $n-k$ equations needed for a solution. Having solved for γ_1 and the unknown elements of γ_0, we then obtain the remaining γ_i coefficients from Equation (1.24).

Alternatively the solution of (1.20) can be obtained directly without forming a large first-order system. By substituting the general solution of y_t into (1.24) and by examining the equation in the Γ_i coefficients, the solution can be obtained by factoring the characteristic polynomial associated with these equations. This approach was used by Hansen and Sargent (1981), where $p = q$ and $B_i = hA_i'$. In that case, the factorization can be shown to be unique by an appeal to the factorization theorems for spectral-density matrices. In general econometric applications, these special properties on the A_i and B_i matrices do not hold. Whiteman (1983) has a proof that a unique factorization exists under conditions analogous to those placed on the roots of A in Equation (1.22). Dagli and Taylor (1984) proposed an iterative method to factor the polynomials in the lag operator in order to obtain a solution. This factorization method is used to solve and estimate the five-equation rational expectations model of the United States using full information maximum likelihood (see Chapter 2).

Nonlinear Models

Unfortunately, many practical rational expectations models are not linear, so that the above methods cannot be used. However, nonlinear solution methods are available and, although computationally different from the above methods, they are conceptually very similar. They can be used to compute the effects of policy in much the same way.

A general nonlinear rational expectations model can be written as

$$f_i(y_t, y_{t-1}, \ldots, y_{t-p}, E_t y_{t+1}, \ldots, E_t y_{t+q}, a_i, x_t) = u_{it} \tag{1.28}$$

for $i = 1, \ldots, n$, where y_t is an n-dimensional vector of endogenous variables at time t, x_t is a vector of exogenous variables, a_i is a vector of parameters, and u_{it} is a vector of disturbances.

Fair and Taylor (1983) proposed an iterative method—called the extended path method—to solve this type of nonlinear model. Briefly it works as follows. Note that if we knew the expectations of future variables, Equation (1.28) would be easy to solve. It would be a standard system of simultaneous equations that could be solved using some nonlinear method, such as Gauss-Siedel, which is the method used to solve conventional (non-rational expectations) models. The solution would provide values for variables y_t. The extended path method works by guessing and successively updating the guesses of these future variables. For each guess, the model is solved, providing an updated guess. The model is solved again and so on.

To be more specific, first guess values for the expectations $E_t y_{t+j}$ in Equation (1.28) for a particular horizon $j = 1, \ldots, J$. Second, using these values, solve the model to obtain a new solution path for y_{t+j}. Third, replace the initial guess of $E_t y_{t+j}$ with the new solution y_{t+j}. The three steps can then be repeated again and again until the solution path y_{t+j} for $j = 1, \ldots, J$ converges.

However, this solution may depend on the horizon J. To check this, extend the solution path from J to $J + 1$ and repeat the previous sequence of iterations until convergence is reached. If the values of y_{t+j} for this extended horizon ($J + 1$) are within the tolerance range of the values for the J-period horizon, then stop; otherwise, extend the path one more period to $J + 2$ and so on. Because the model is nonlinear, the Gauss-Seidel method is used for each iteration given a guess for the expectation variables.

There are no general proofs available to show that this method works for an arbitrary nonlinear model. When applied to the linear model in Equation (1.1) with $|\alpha| < 1$, the method converges as demonstrated by Fair and Taylor (1983). When $|\alpha| > 1$, the iterations diverge. A convergence proof for the general linear model is not available, but many experiments have indicated that convergence is achieved under the usual assumptions.

The extended path method is fairly easy to use and has become the most common method of solving large nonlinear rational expectations models. It is used extensively in this book to solve the nonlinear multicountry model.

Note that once a solution is obtained, the stochastic properties of y_t can be determined by stochastic simulations: different values for u_{it} on the right-hand side of Equation (1.28) can be drawn from a random-number generator. The means, variances, and covariances of the elements of y_t can then be calculated by averaging across these draws. Moreover, the effects of one-time changes in the instruments are obtained by deterministic simulation, that is, by solving the model for a different path of the exogenous variables. The effects of anticipated and unanticipated changes in the instrument can both be calculated. Hence, although the method is numerical rather than analytical, all the policy simulations conducted with a linear model can be conducted with a nonlinear model.

1.3 The Lucas Critique

Lucas (1976) argued that the parameters of the models conventionally used for policy evaluation—either through model simulation or formal optimal control—would shift when policy changed. The main reason for this shift is that expectations mechanisms are adaptive, or backward looking, in conventional models and thereby unresponsive to those changes in policy that would be expected to change expectations of future events. Hence, the policy-evaluation results using conventional models would be misleading.

The Lucas criticism of conventional policy evaluation has typically been taken as destructive. Yet, implicit in the Lucas criticism is a constructive way to improve on conventional evaluation techniques by modeling economic phenomena in terms of "structural" parameters; by "structural," one simply means invariant with respect to policy intervention. Whether a parameter is invariant or not is partly a matter of a researcher's judgment, of course, so that any attempt to take the Lucas critique seriously by building structural models is subject to a similar critique that the researcher's assumption about which parameters are structural is wrong. This applies even if the only structural parameters are the "deep parameters" of utility functions. If taken to this extreme that no feasible structural modeling is possible, the Lucas critique does indeed become purely destructive and perhaps even stifling. The three examples used by Lucas in his critique were a Friedman-type consumption equation, a Hall-Jorgenson investment equation, and a Phillips curve. None of the examples incorporated the deep parameters of utility functions, although all could clearly benefit—and have benefited—from greater theoretical research. So could the money-demand equation used here for illustration.

Consider the following policy problem, which is based on a model like that of Equation (1.3). Suppose that an econometric policy advisor knows that the demand for money is given by

$$m_t - p_t = -\beta(E_t p_{t+1} - p_t) + u_t. \tag{1.29}$$

Here there are two shocks to the system, the supply of money m_t and the demand for money u_t. Suppose that $u_t = \rho u_{t-1} + \varepsilon_t$ and that in the past the money supply was fixed: $m_t = 0$; suppose that under this fixed money policy, prices were thought to be too volatile. The policy advisor is asked by the Central Bank to advise on how m_t can be used in the future to reduce the fluctuations in the price level. Note that the policy advisor is not asked just what to do today or tomorrow, but what to do for the indefinite future. Advice thus should be given as a contingency rule rather than as a fixed path for the money supply.

The behavior of p_t during the past is

$$p_t = \rho p_{t-1} - \frac{\varepsilon_t}{1 + \beta(1 - \rho)}. \tag{1.30}$$

Conventional policy evaluation might proceed as follows: first, the econometrician would have estimated ρ in the reduced-form relation (1.30) over the sample period. The estimated equation would then serve as a model of expectations to be substituted into (1.31); that is, $E_t p_{t+1} = \rho p_t$ would be substituted into

$$m_t - p_t = -\beta(\rho p_t - p_t) + u_t. \tag{1.31}$$

The conventional econometrician's model of the price level would then be

$$p_t = \frac{m_t - u_t}{1 + \beta(1 - \rho)}. \tag{1.32}$$

Considering a policy rule of the form $m_t = g u_{t-1}$, Equation (1.32) implies

$$\operatorname{Var} p_t = \frac{1}{[1 + \beta(1 - \rho)]^2 (1 - \rho^2)} \sigma_\varepsilon^2 [g^2 + 1 - 2g\rho]. \tag{1.33}$$

Equation (1.33) indicates that the best choice for g to minimize fluctuation in p_t is $g = \rho$.

But we know that Equation (1.33) is incorrect if $g \neq 0$. The error was to assume that $E_t p_{t+1} = \rho p_t$ regardless of the choice of policy. This is precisely the point of the Lucas critique. The correct approach would have been to substitute $m_t = g u_{t-1}$ directly into Equation (1.29) and to calculate the stochastic process for p_t. This results in

$$p_t = \frac{-1 - \beta(1 - g)}{(1 + \beta)(1 + \beta(1 - \rho))} u_t + \frac{g}{1 + \beta} u_{t-1}. \tag{1.34}$$

Note how the parameters of Equation (1.34) depend on the parameters of the policy rule. The variance of p_t can thus easily be calculated, and the optimal policy is found by minimizing $\operatorname{Var} p_t$ with respect to g.

This simple policy problem suggests the following approach to macropolicy evaluation: (1) derive a stochastic equilibrium solution that shows how the endogenous variables behave as a function of the parameters of the policy rule; (2) specify a welfare function in terms of the moments of the stochastic equilibrium; and (3) maximize the welfare function across the parameters of the policy rule. In this example, the welfare function is simply Var p.

The Lucas critique can be usefully thought of as a dynamic extension of the critique developed by the Cowles Commission researchers in the late 1940s and early 1950s, which gave rise to the enormous literature on simultaneous equations. At that time it was recognized that reduced forms could not be used for many policy-evaluation questions. Rather, one should model structural relationships. The parameters of the reduced form are, of course, functions of the structural parameters in the standard Cowles Commission setup. The discussion by Marschak (1953), for example, is remarkably similar to the more recent rational expectations critiques; Marschak did not consider expectations variables, and in this sense, the rational expectations critique is a new extension. But earlier analyses like Marschak's were an effort to explain why structural modeling is necessary, and thus they have much in common with more recent research.

1.4 Economic Policy Rules and Shocks in a Stylized Two-Country Model

The previous two sections showed how rational expectations models can be used to calculate the effects of one-time changes in the policy instruments and to evaluate the properties of different policy rules. The primary example, however, has been very simple: a one-policy variable, such as the money supply, and a one-target variable, such as the price level.

To illustrate how the method works in a more meaningful setting, this section examines the effects of policy in a stylized two-country model. The model is similar to that found in an undergraduate textbook model, except that it presents rational expectations and staggered wage setting that generate both short-run fluctuations in the economy and long-run neutrality. In addition, the two countries are linked together by a capital market with perfect capital mobility. In the same way that models used in most undergraduate texts provide a stylized account of how traditional econometric models without rational expectations work, this section provides a stylized account of how the econometric models with rational expectations work. Understanding how the model works will aid greatly in understanding the more complex econometric models introduced later in the book.

Table 1-1 displays the equations of the model and defines the notation. There are two countries: A and B. All the variables except the interest rates and the inflation rates are measured as logarithms, and all variables are deviations from means or secular trends. For example, y is the deviation of the log of real output from secular or potential output. Potential output is assumed to be unaffected by the policy changes considered here, although that assumption could be modified. Equations (1A) through (6A) in Table 1-1 describe country A; Equations (1B) through (6B) describe country B; an

TABLE 1-1 Stylized Two-Country Rational Expectations Model

Country A

$$x_t = (\delta \Rightarrow 3) \sum_{i=0}^{2} \hat{w}_{t+i} + [(1-\delta) \Rightarrow 3] \sum_{i=0}^{2} \hat{p}_{t+i} + (\gamma \Rightarrow 3) \sum_{i=0}^{2} \hat{y}_{t+i} \tag{1A}$$

$$w_t = \frac{1}{3} \sum_{i=0}^{2} x_{t-i} \tag{2A}$$

$$p_t = \theta w_t + (1-\theta)(e_t + p_t^*) \tag{3A}$$

$$y_t = -dr_t + f(e_t + p_t^* - p_t) + gy_t^* \tag{4A}$$

$$m_t - p_t = -bi_t + ay_t \tag{5A}$$

$$r_t = i_t - \hat{\pi}_t \tag{6A}$$

Capital Mobility Condition

$$i_t = i_t^* + \hat{e}_{t+1} - e_t$$

Country B

$$x_t^* = (\delta^* \Rightarrow 3) \sum_{i=0}^{2} \hat{w}_{t+i}^* + [(1-\delta^*) \Rightarrow 3] \sum_{i=0}^{2} \hat{p}_{t+i}^* + (\gamma^* \Rightarrow 3) \sum_{i=0}^{2} \hat{y}_{t+i}^* \tag{1B}$$

$$w_t^* = \frac{1}{3} \sum_{i=0}^{2} x_{t-i}^* \tag{2B}$$

$$p_t^* = \theta^* w_t^* + (1-\theta^*)(p_t - e_t) \tag{3B}$$

$$y_t^* = -d^* r_t^* - f^*(e_t + p_t^* - p_t) + g^* y_t \tag{4B}$$

$$m_t^* - p_t^* = -b^* i_t^* + a^* y_t^* \tag{5B}$$

$$r_t^* = i_t^* - \hat{\pi}_t^* \tag{6B}$$

Definition of Variables and Parameter Values

Variables

y_t = real GNP (log)
p_t = price level (log)
i_t = nominal interest rate
r_t = real interest rate
π_t = inflation rate
w_t = nominal wage (log)
m_t = money supply (log)
x_t = contract wage (log)
e_t = exchange rate (log); country A price of country B currency
$\hat{}$ = conditional expectation based on information through period t

Parameter values for simulations

$\delta = 0.5$
$\gamma = 1.0$
$\theta = 0.8$
$d = 1.2$
$f = 0.1$
$g = 0.1$
$b = 4.0$
$a = 1.0$

asterisk denotes the variables of country B. The remaining equation is the condition of perfect capital mobility: the interest rate in country A is equal to the interest rate in country B plus the expected rate of depreciation of the currency of country A. In a Mundell-Fleming model with fixed prices and no expectations, the interest rates are equal. Because the structure in the two countries is the same, we need to describe only the equations in country A.

Equation (1A) is a staggered wage-setting equation much like that used in Taylor (1980). The "contract" wage is denoted by (x). A wage decision is assumed to last for three periods, with only one-third of the wages being negotiated in any one year. The wage set at time t depends on expectations of future wages paid to other workers, expectations of prices, and expectations of future demand conditions as proxied by the deviation of real output from trend. Equation (2A) defines the average wage in the economy as a whole. Equation (3A) is a markup pricing equation; prices of domestic goods are a weighted average of wages and the prices of imported inputs to production measured in domestic currency units. Equations (4A) and (5A) are *IS* and *LM* curves respectively, just like those found in undergraduate texts. The real interest rate differs from the nominal interest rate according to the rationally expected inflation rate as described in Equation (6A).

The Impact of Changes in the Policy Instruments

The Closed Economy. To give some perspective to the two-country results, first consider the effects of changes in the instruments of monetary and fiscal policy in the *closed* economy described by Equations (1A) through (6A) with $\theta = 1, f = 0$, and $g = 0$. These restrictions correspond to no international linkages. These are the types of experiments one needs to run to find out the properties of this kind of model. As we will see, they are also useful in studying the transition from one policy rule to another. The coexistence of rational expectations and forward-looking, though sticky, prices gives rise to a number of phenomena that are unlike standard models.

Consider separately a money shock and a fiscal shock.[3] The money shock is a 1-percent unanticipated permanent increase in the money supply, and the fiscal shock is a 1-percent unanticipated permanent rightward shift in the *IS* curve (Equation [4A]). The latter shift could be due to a change in government purchases. The results are shown in Figure 1-2. The figure shows the actual values of the variables rather than their logarithms. In Figure 1-2, the fiscal shock is denoted by a dashed line, and the money shock is denoted by a solid line. If only a solid line appears for a particular period, the effects of the money and fiscal shocks are the same. No attempt

[3]The model was solved using the extended path algorithm described in Section 1.2. Alternatively, since the model is linear, it could be solved by the factorization algorithm or by computing the roots explicitly as described in Section 1.2, although for the higher-order models this might not be practical.

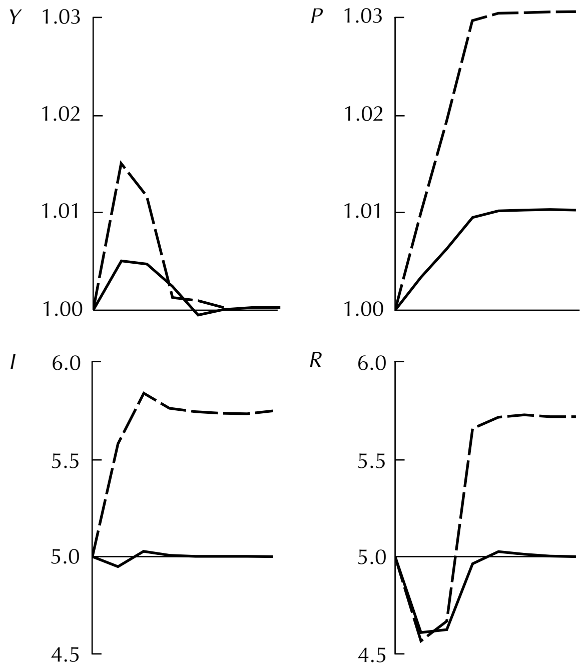

FIGURE 1-2 Policy Impact in a Closed Economy. The chart shows the impact of fiscal (dashed line) and monetary (solid line) shocks in a closed economy on real output (Y), price level (P), nominal interest rate (I), and real interest rate (R).

has been made to scale the shocks so as to give similar effects for monetary and fiscal policy.

Monetary policy has an expected positive effect on output that dies out as prices rise and real-money balances fall back to where they were at the start. Note that the real interest rate drops more than the nominal rate because of the increase in expected inflation that occurs at the time of the monetary stimulus. For this set of parameters the nominal interest rate hardly drops at all; all the effect of monetary policy shows up in the real interest rate.

Fiscal policy creates a similar dynamic pattern for real output and for the price level. Note, however, that there is a surprising "crowding-in" effect of fiscal policy in the short run as the increase in the expectation of inflation causes a drop in the real interest rate. Eventually the expected rate of inflation declines and the real interest rate rises; in the long run, private spending is completely crowded out by government spending.

Two Countries with a Flexible Exchange Rate. The effects of monetary and fiscal shocks in the full two-country model are shown in Figure 1-3 when the exchange rate is perfectly flexible. For these simulations the parameters are assumed to be the same in both countries and are given in Table 1-1. In all of

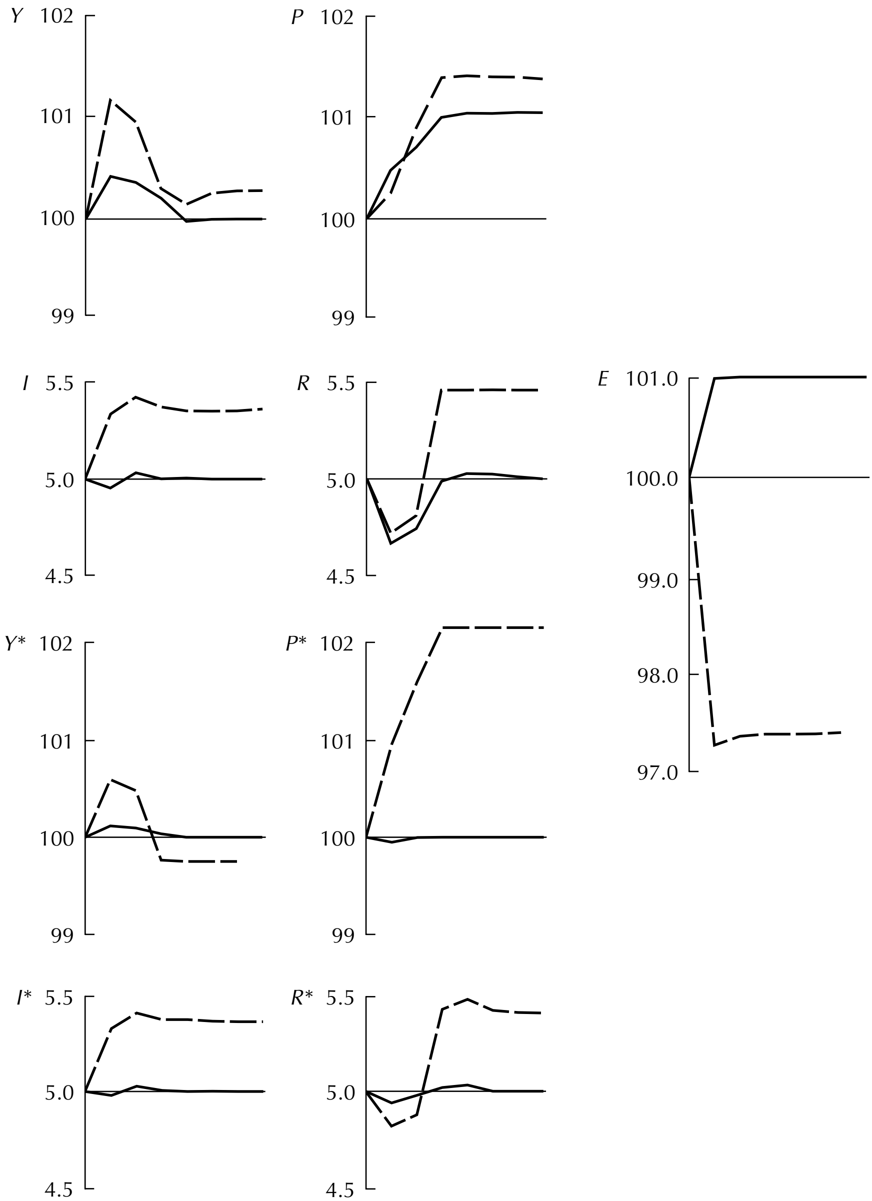

Figure 1-3 Policy Impacts in a Two-Country Model with Flexible Exchange Rates. The charts show the effects of fiscal (dashed line) and monetary (solid line) shocks.

these experiments, the policy shock occurs in country A. Later in the book we conduct such experiments in an empirically estimated multicountry model.

The dynamic impact in country A of a fiscal shock is similar to the closed-economy case. The initial impact on real output is only slightly less than in the closed economy, and the effect dies out at about the same rate. There is also an initial drop in the real interest rate, and this is the primary reason for the strong effect of fiscal policy in the flexible exchange-rate regime. As in the fixed-price Mundell-Fleming model, the exchange rate of country A appreciates, so that exports are crowded out by fiscal policy, but the drop in the real interest rate stimulates investment. Note that the long-run output effect of the fiscal shock is slightly positive in country A. This is matched by an equally negative long-run output effect in country B. However, there is an initial positive output effect in country B as the real interest rate first declines before increasing and crowding out investment spending. Fiscal policy has inflationary effects abroad, partly because of the depreciation of the foreign currency.

The effect of an increase in the money supply in country A is also much like that in the closed economy. There is a positive short-run effect on output that diminishes to zero over time. Part of the monetary stimulus comes from a depreciation of the currency of country A, and part comes from the decline in real interest rates. There is no significant overshooting of the exchange rate following the monetary impulse. Unlike in the Mundell-Fleming model, however, the increase in the money supply is not contractionary abroad. A monetary stimulus can have a positive effect abroad because the price level is not fixed; the depreciation of country A's currency reduces prices in country B, and this raises real balances in that country. The real interest rate also declines slightly in country B.

Two Countries with a Fixed Exchange Rate. For comparison we report in Figure 1-4 the results from similar experiments with fixed exchange rates. For this purpose, the model is altered; the exchange rate becomes an exogenous variable, and the money supply in country B becomes an endogenous variable. The capital-mobility condition is then simply $i_t = i_t^*$. Again the shocks occur in country A. But now country B must give up an independent monetary policy. The money supply in country B must move around in order to keep the exchange rate fixed.

The short-run output effects of fiscal policy with fixed exchange rates are a bit weaker in country A compared with the flexible exchange-rate case. The output effects abroad are strongly negative, even in the short run. There is no short-run decline in the real interest rate in country B, as there was when the exchange rate could adjust. In fact, the real interest rate in country B overshoots its new higher long-run equilibrium value. Note that in order to keep the exchange rate fixed, country B must reduce its money supply. This means that its price level must eventually fall; in the

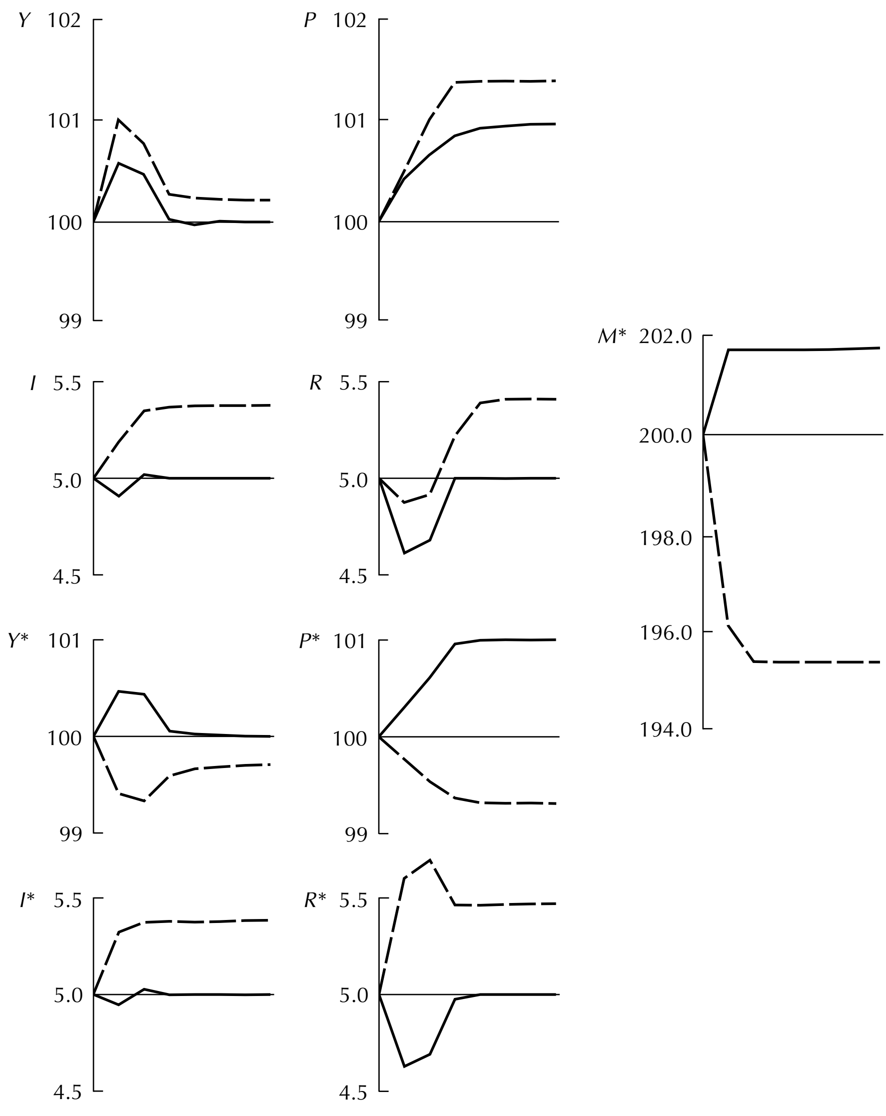

Figure 1-4 Policy Impacts in a Two-Country Model with Fixed Exchange Rates. The charts show the effects of fiscal (dashed line) and monetary (solid line) shocks.

short run there is thus an expected deflation that raises the real interest rate in country B for a time above the long-run equilibrium.

Monetary policy has a larger effect on real output in country A than in the flexible exchange rate case. In the long run, the output effect diminishes, and the price level rises by the same amount as the money supply. The effect of this monetary policy on the other country is much stronger than in the case of flexible exchange rates. In order to keep the exchange rate fixed, the monetary authority in country B must expand its money supply by the same amount as the money increase in country A. This has stimulative effects on real output that duplicate the effects of money in country A.

Effects of Changes in Policy Rules

The problem of designing policy rules can also be illustrated in this stylized model. This is an issue that will be considered empirically in Chapters 2 and 6. There are obviously many alternative policy rules to consider. Besides policy rules for the money supply, one can consider policy rules for interest rates. For example, a nominal interest-rate rule could take the form $i_t = a_i p_t$. A real interest-rate rule could have a similar form $r_t = a_r p_t$. Both of these are different from a money-supply rule of the form $m_t = a_m p_t$. All three are possible characterizations of monetary policy. These rules state that the policy instruments should be changed whenever prices rise above target. Recall that in this model the price target is normalized to zero. The effects of these rules can be calculated by plugging them into the model and by solving the model.

The real interest-rate rule for the single-country model ($\theta = 1, f = 0, g = 0$) is particularly easy to analyze. Such a rule can be substituted directly into Equation (4A) to obtain an equation involving p_t and y_t (an aggregate-demand equation). Combining this with Equations (1A), (2A), and (3A) gives a simple two-variable model from which stochastic processes for p_t and y_t can be solved. In fact, that model is exactly the same as the simple staggered contract model of Taylor (1980). By varying parameter a_r of the policy rule, the "operating characteristics" of prices and output change. A trade-off between the variance of output and the variance of the price level is traced out.

Consider the variance of output and prices in the two-country world economy under alternative real interest-rate rules. Since there are two countries, we need to specify two such interest-rate rules. Let these be:

$$\begin{aligned} r_t &= a_r p_t, \\ r_t^* &= a_r^* p_t^*. \end{aligned}$$

We can solve and stochastically simulate the two-country model for different values of a_r and a_r^*. Variances calculated for policy rules for four cases in which a_r and a_r^* equal 0.2 and 0.6 are reported in Table 1-2. These calculations are made under the assumption that only supply shocks continually occur in both countries, that these shocks are unanticipated and temporary, and that they are uncorrelated between the countries. In other words, Equations (1A) and (1B) are continuously shocked by serially and contemporaneously uncorrelated random variables.

Table 1-2 indicates that there is relatively little interaction between the policy rules in the two countries. For example, as the home country moves

Table 1-2 Output and Price Variability in Two Countries with Alternative Policy Rules

	$\alpha_r^* = 0.6$	$\alpha_r^* = 0.2$
$\alpha_r = 0.6$	0.188 0.147 0.188 0.147	0.423 0.111 0.181 0.144
$\alpha_r = 0.2$	0.181 0.144 0.423 0.111	0.425 0.112 0.425 0.112

Key	
	σ_p^* σ_y^* σ_p σ_y

from a relatively nonaccommodative interest rate rule to a more accommodative one, its output variability declines, and its price variability increases. But the effect of this move on the other country's variability measure is very small.

An important question is whether these results are also true in more realistic, empirical models. Such a model is developed in Chapter 3 of this book and is used in the remaining chapters to examine this question and many others. But before that we take a first look at econometric policy evaluation in the next chapter.

Reference Notes

The brief discussion of policy rules in Section 1.1 only touches on a very large literature. A useful review of the definition of policy rules, including Friedman's (1948) proposal, and of the rules-versus-discretion debate is found in Fischer's (1990) *Handbook of Monetary Economics* paper. In my view the Kydland and Prescott (1977) work is still the best source on time inconsistency in macroeconomics and is well worth reading; Barro and Gordon (1983) introduced different, perhaps less confusing, terminology for Kydland and Prescott's different solution concepts and also studied reputation as means to maintain the "rules" or the "optimal" solution. The Blanchard and Fischer (1989) text provides a comprehensive review of the Kydland and Prescott models and follow-up models. I am not aware of other work that has explicitly made the distinction between design, transition, and operation of policy rules, though it seems like a natural distinction and is implicit in many discussions. A study of learning during the transition from one policy rule to another is found in Taylor (1975).

The method introduced in Section 1.2 for solving linear rational expectations models is found and explained more fully in my *Handbook of Econometrics* paper (Taylor, 1986). An introduction to dynamic stochastic models needed for this method is provided in Chow (1975) and can now be found in most econometrics texts. Sargent's (1987a) macroeconomics text provides a comprehensive treatment of stochastic difference equations with applications to macroeconomics. Factorization methods for multivariate linear systems and the expended path method are also discussed in the *Handbook* paper (Taylor, 1986). The particular method of undetermined coefficients used in Section 1.2 was the one used by Muth (1961) in his original paper on rational expectations.

The best background reading on the Lucas critique is the original Lucas (1976) paper on the subject. Section 1.3 comes close to illustrating how the critique is dealt with in this book: by plugging alternative rules into model economies, seeing how they work, and informally searching for the optimal rule. More formal methods to find the optimal policy rule in rational expectations models can be found in Taylor (1979), in Hansen, Epple, and Roberds (1985), and in Sargent (1987b).

The stylized two-country model in Section 1.4 was introduced in Carlozzi and Taylor (1985) and in Taylor (1985). References to the two-country Mundell-Fleming model, upon which this model builds, are Mundell (1962) and Fleming (1962). Dornbusch (1976) first introduced rational expectations into a single-country Mundell-Fleming model with capital mobility and focused on the question of exchange rate overshooting, which is slightly evident in Figure 1-3.